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The Becker-DeGroot-Marschak mechanism is not necessarily incentive compatible, even for non-random goods

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Abstract

This paper uses results from [Karni, E., Safra, Z., 1987. Preference reversals and the observability of preferences by experimental methods, Econometrica 55, 675–685] and [Karni, E., 1989. Generalized expected utility analysis of multivariate risk aversion, International Economic Review 30, 297–305] to show that the Becker-DeGroot-Marschak mechanism is not always incentive compatible, even when the item to be valued involves no uncertainty, contrary to several recent assertions.

Keywords: Becker-DeGroot-Marschak; Incentive compatibility; Valuation; Vickrey auction; Willingness-to-pay

JEL classification: D82; Q51

1. Introduction

Economists are often interested in knowing how much an individual would pay for some item. The Becker-DeGroot-Marschak mechanism (BDM) is a common method for eliciting this willingness-to-pay (Becker et al., 1964). Under the BDM, an individual reports a bid for an item; the item's price is then randomly drawn. If the bid is above the price, the individual receives the good and pays the drawn price. If the bid is below the price, the individual does not receive the good and pays nothing.

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This mechanism is simple and presumed to induce truth-telling. Individuals have the incentive, it is believed, to report their true maximum willingness-to-pay. This incentive is supposedly "robust" in the sense that truth-telling is a dominant strategy and therefore independent of risk attitudes and even of whether the individual is an expected utility maximizer. Recent claims that the BDM is incentive compatible (without conditions) for non-random goods can be found in Davis and Holt (1993, p.461), Kahneman et al. (1990, p. 1336), Rutstrõm (1998, p. 428), and Shogren et al. (2001, p. 98).

This presumption about the BDM is false. In an important paper, Karni and Safra (1987) showed that the BDM is not incentive compatible when the object being valued is a lottery. The applicability of their results to the case of non-random goods has not been recognized. This paper shows that the BDM is not incentive compatible even when the item to be valued involves no uncertainty.

The argument is as follows. The value an individual places on an item is not independent of the circumstances in which he is asked to pay for it. Under the BDM, the individual is uncertain about how much he will be asked to pay. Thus, his willingness-to-pay would reasonably be expected to depend on the distribution of potential prices. Thus, the individual's optimal bid when he faces an unknown price need not be the same as the highest price at which he would buy the item if his choice was whether to buy the good at a known price. This latter is typically referred to as "true" willingness-to-pay.

There is another simple way to frame this argument. After an individual reports his bid, he still faces uncertainty: uncertainty over whether his bid will be accepted and what the price will be. Given that the nature of this uncertainty is determined in part by his bid, it follows that his bid may depend on the distribution of potential prices.

The reason why the BDM has been presumed to be incentive compatible appears to be the same trap that researchers fell into for lotteries prior to Karni and Safra. If the utility for an item is summarized by its value under certainty, then the BDM does induce truth-telling, a result based only on dominance. This trap has led researchers to wrongly think that a random price was irrelevant for items that involved no uncertainty. However, in general it is wrong to summarize preferences with their certainty equivalent.

This non-incentive-compatibility result also holds for the Vickrey auction and general nth-price auctions. These auctions also are not always incentive compatible, even for non-random goods. The argument is the same as for the BDM. Because the potential price of the item is random, an individual's bid is potentially affected by the distribution of prices. Therefore, the individual might not bid his true cut-off price. We focus here on the BDM, however, because the auction literature has more often explicitly recognized the dependence of results on risk-neutrality or expected-utility maximization (e.g., Kagel, 1995; Milgrom and Weber, 1982).

2. Model and result

Let $q=\{0,1\}$ be the amount of the good, which is assumed to be exogenous to the individual. In a typical experiment, an individual starts with $q=0$ and is asked the most he would be willing to pay to receive $q=1$. Let Y be his income.

Under the BDM, the price of the item is unknown to the subject. He reports a value, v. The price c is then revealed. If v is greater than the revealed price, the individual gets the item and pays c. If v is less than the revealed price, the individual does not get the item and pays nothing.

From the individual's point of view, this price is random when he reports v. Let $F(\cdot)$ be the distribution of the price. Under the BDM, the individual receives $\{q=1, Y-c\}$ whenever c is less than v;

this state occurs with probability $F(v)$. The individual receives $\{q=0,Y\}$ whenever c is greater than v; this state occurs with probability $1-F(v)$. Define this induced (multivariate) distribution of q and the realized price (either c or 0) as F_v.

Under expected utility (EU), utility is:

$$U(F_v) = \int_0^v u(1, Y-c)\mathrm{d}F(c) + u(0,Y)(1-F(v)) \tag{1}$$

The individual reports v to maximize utility. The optimal report, v^*, satisfies:

$$u(1, Y-v^*) = u(0,Y) \tag{2}$$

To define incentive compatibility in this context, consider the situation where the individual faces a dichotomous choice with known c. The individual would agree to pay c and receive $q=1$ if and only if $u(1,Y-c)\geq u(0,Y)$. A report v^{IC} is *incentive compatible* – that is, it is the maximum amount the individual would pay under certainty or, equivalently, the minimum price at which he would agree to purchase the good – if and only if $u(1,Y-c)\geq u(0,Y)$ for all $c\leq v^{\mathrm{IC}}$. Thus, v^{IC} solves $u(1,Y-v^{\mathrm{IC}})=u(0,Y)$. Therefore, the BDM is incentive compatible under EU.

When preferences are smooth but not necessarily EU, as in Machina (1982), the first-order condition analog to Eq. (2), defining the optimal report, is:

$$u(1, Y-v^*; F_{v^*}) = u(0, Y; F_{v^*}) \tag{3}$$

where $u(q,y;F)$ is the local utility function. In words, v^* depends on F. This is the key result.

It should now be clear from both intuition and Eq. (3) that the BDM is not in general incentive compatible. Incentive compatibility means that the individual reports the minimum (non-random) price at which he would purchase the good. It thus is defined based on preferences over degenerate lotteries. Preferences induced by local utility functions do not, however, in general correspond to preferences over degenerate lotteries in the multivariate case, as pointed out by Karni (1989).[1]

3. An example using disappointment-aversion

This section gives an example of NEU preferences where the BDM is not incentive compatible. The example is a variant of Disappointment Aversion, originally proposed by Gul (1991); the example here also builds on a direction suggested by Routledge and Zin (2003). This model is an intuitively appealing depiction of individuals' responses to a BDM or nth-price auction, but I do not claim that it is the "right" model of BDM or auction behavior.

For any given report v, let $\bar{c}(v)$ be the certainty equivalent price for the resulting lottery; for ease of notation, let this be $\bar{c}$. Conditional on reporting v, an individual will be said to be disappointed when the actual price c is either (i) above $\bar{c}$ but below v, in which case the individual gets the good and pays c, but this is more than his certainty equivalent price; or (ii) above v, in which case the individual does not get

[1] Karni (1989) in turn builds on Kihlstrom and Mirman (1974). Safra and Segal (1993) derive conditions under which local utility functions are ordinally equivalent, the condition required for incentive compatibility.

the good. Suppose that these two types of disappointment are viewed differently by the individual, with penalties β and γ, respectively. The certainty equivalent is given implicitly by:

$$u(1, Y-\bar{c}) = \int_0^v u(1, Y-c)\mathrm{d}F(c) + u(0, Y)(1-F(v)) - \beta\int_{\bar{c}}^{v}[u(1, Y-\bar{c}) - u(1, Y-c)]\mathrm{d}F(c) - \gamma[u(1, Y-\bar{c}) - u(0, Y)](1-F(v)). \tag{4}$$

Note that if $\beta=\gamma$ this is Gul's model applied to the BDM. If $\{\beta=0, \gamma>0\}$, this is close to Routledge and Zin's model in that only "extreme" results are disappointing.

The parameters β and γ depend on the value of the good and the distribution of prices. Suppose the range of prices tends to be low relative to the individual's value for the good, so that the individual expects to get positive utility from this mechanism. A natural assumption in this case is that he will be more disappointed if he does not get the good; that is, $\gamma>\beta$. Note that β and γ do not depend on the individual's reported value, so it is possible to hold β and γ constant while varying the report v.

To see the optimal report in relation to v^{IC}, differentiate the right-hand-side of Eq. (4) and evaluate at v^{IC}: $\mathrm{d}U(v)/\mathrm{d}v=\partial U/\partial v+(\partial U/\partial \bar{c})\mathrm{d}\bar{c}/\mathrm{d}v$, where U is the NEU functional. This step yields:

$$\left.\frac{\mathrm{d}U}{\mathrm{d}v}\right|_{v^{\mathrm{IC}}} = \frac{(\gamma-\beta)[u(1, Y-\bar{c}) - u(0, Y)]f\left(v^{\mathrm{IC}}\right)}{1+\beta[F(v^{\mathrm{IC}}) - F(\bar{c})] + \gamma[1-F(v^{\mathrm{IC}})]} \tag{5}$$

Suppose $\gamma>\beta$. Then the expression in Eq. (5) is positive, which implies $v^*>v^{\mathrm{IC}}$. The individual reports a value higher than the true one. This occurs because $\gamma>\beta$ implies that the individual is more disappointed from not receiving the good than from receiving it but having to pay a relatively high price. Thus, she reports a higher value to increase the chance of getting the good.

Note that the BDM is incentive compatible for two common versions of NEU preferences. It is incentive compatible under standard disappointment aversion, $\beta=\gamma$; in this case, both forms of disappointment receive equal weight, so the optimal report is to give the true ex post value.

The BDM is also incentive compatible for rank dependent expected utility (RDEU). Under RDEU, the utility for report v is:

$$\int_0^v u(1, Y-c)h(F)\mathrm{d}F(c) + \int_v u(0, Y)h(F)\mathrm{d}F(c) \tag{6}$$

where $h(F)$ is the weighting function derivative. Differentiation over v yields $u(1,Y-v^*)=u(0,Y)$, and the BDM is incentive compatible.

4. Why the BDM has been presumed to be incentive compatible

A demonstration of why the BDM has been presumed to be incentive compatible, based on a dominance argument, can be shown as follows. Define a local utility function $\hat{u}(y;F)$ in which preferences for the good are captured as:

$$\hat{u}\left(y+v^{\mathrm{IC}}q;F\right) \tag{7}$$

In words, the utility of q has been summarized by its money-denominated value under certainty, v^{IC}. When $q=1$, utility is $\hat{u}(y+v^{IC};F)$. When $q=0$, utility is $\hat{u}(y;F)$.

Note that for any $c<v^{IC}$, we have $\hat{u}(Y+v^{IC}-c;F)>\hat{u}(Y;F)$. Dominance (alone) implies that any increase in the probability of the former state at the expense of the latter state is desirable for this utility function. Under the BDM, this probability is increased by increasing the reported valuation, v. When $c>v^{IC}$, an increase in the probability of the latter state is desirable and this probability is decreasing in v. Thus, utility is maximized when $v=v^{IC}$. This claim holds for all distributions F and for all increasing utility functions.

Thus, the BDM and its more common counterpart, the nth-price auction, are incentive compatible when preferences are correctly captured by Eq. (7). Eq. (7), however, does not correctly represent preferences in general.

This approach, which appears to be the way that most researchers have conceived of the BDM, makes the same mistake that early researchers made when studying lotteries. In Eq. (7), the good has been replaced with its certainty value. The BDM does give the correct incentive to report "certainty value". But the certainty value is not independent of the distribution of prices, so such an incentive is irrelevant. As with choice over lotteries, it is wrong to replace the good with its certainty equivalent except in the case of EU preferences, a result demonstrated by Karni and Safra (1987).

Researchers will also want to know whether any of these effects is large enough to matter in real-world applications of the BDM or auctions. We take this topic up in a separate paper.

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